DISTORTION OF A TURBULENT-VISCOSITY PROFILE BY LARGE NEGATIVE PRESSURE GRADIENTS IN GAS FLOW THROUGH PIPES

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On the basis of tests performed at the Grenoble University, the profiles of turbulent viscosity are calculated as functions of the negative pressure gradient.

The study concerns developed turbulent flow of a gas through pipes under a "critical" condition characterized by a very rapidly increasing negative pressure gradient in the vicinity, $dp/dx \to -\infty$. All quantities here are dimensionless: the velocity is referred to the limiting velocity, the temperature is referred to the stagnation temperature, the pressure and the density are referred to the entrance ($x = 0$) pressure and density respectively. The dynamic and the turbulent viscosity are both referred to the dynamic viscosity at the pipe wall. The transverse coordinates are referred to the pipe radius, the longitudinal coordinates are referred to the pipe diameter.

Among many tests concerned with this kind of flow, most thorough were those performed at the Grenoble University [1]. As one result of that study, it has been established that negative pressure gradients distort the velocity profile appreciably. Thus, the axial velocity profile becomes flatter as the exit section of a pipe is approached. This profile (except for small regions near the axis and near the wall) is closely enough approximated by the power law

$$u = u_1 y^n, \tag{1}$$

where exponent n decreases fast toward the exit section and becomes approximately 1/15 at a Reynolds number $\mathrm{Re} \cong 5 \cdot 10^5$. The Reynolds number is based here on the pipe radius and on the viscosity at the stagnation temperature. R. Depassel [1] has kindly sent us, upon request, the original data which, together with certain limiting ratios characterizing the critical mode, yield information about the skin-friction characteristics [2]. It has been shown in [2] that, if distortion of the velocity profile is taken into account, the magnitude of τ_0 will increase with a higher Mach number and especially fast near the critical mode.

According to the data in [1], one can find approximately not only the magnitude of τ_0 but also the turbulent viscosity profile ε_τ as a function of the negative pressure gradient. The shearing stress is expressed by the following relation:

$$\tau = (\mu + \varepsilon_\tau) \frac{\partial u}{\partial y}, \tag{2}$$

where τ is determined from the following approximation:

$$\tau = (1-y)\left[f + (\tau_0 - f)(1-y)^m\right]. \tag{3}$$

This approximation satisfies the boundary conditions $\tau|_{y=0} = \tau_0$, $\tau|_{y=1} = 0$. The quantities f and m in (3) have been selected so as to account for the edge effects at the wall and at the axis of a pipe. From the conditions at the wall

$$\left(\frac{\partial \tau}{\partial y}\right)_0 = \tau_0 + \widetilde{\mathrm{Re}}\, \frac{k-1}{2k} \frac{dp}{dx} \tag{4}$$

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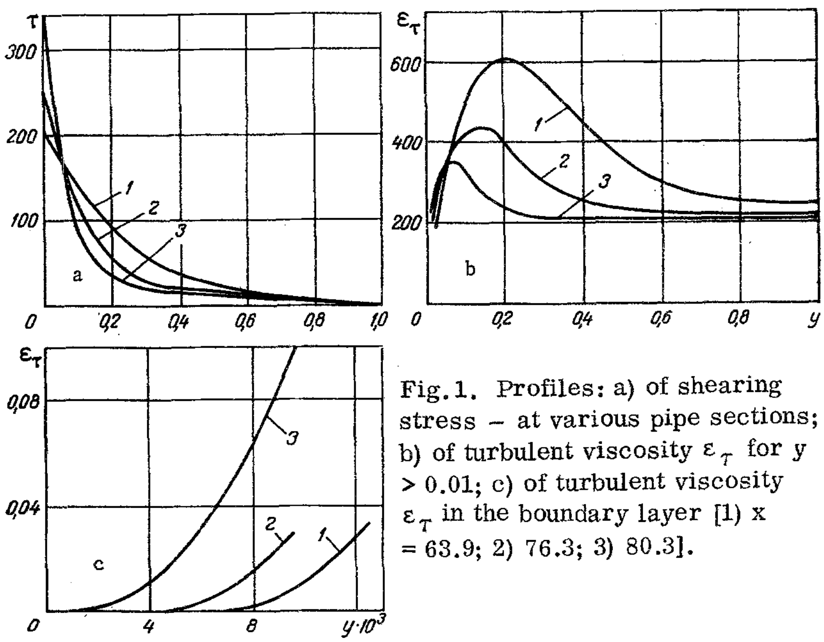

Fig. 1. Profiles: a) of shearing stress – at various pipe sections; b) of turbulent viscosity ε_T for $y > 0.01$; c) of turbulent viscosity ε_T in the boundary layer [1) $x = 63.9$; 2) 76.3; 3) 80.3].

we have for the exponent

$$m = -2\,\frac{\tau_0 + \frac{1}{2}\widetilde{\mathrm{Re}}\,\bar{k}\,p'}{\tau_0 - f},$$

where

$$\bar{k} = \frac{k-1}{2k}\,; \quad p' = \frac{dp}{dx}\,.$$

In the limiting case of an incompressible fluid $\tau_0 + (1/2)\widetilde{\mathrm{Re}}\,\bar{k}p' = 0$, $m = 0$, and relation (3) becomes an ordinary linear one $\tau = \tau_0(1-y)$. Parameter $f = -(\partial\tau/\partial y)$ can be determined from the equation of flow at the pipe axis:

$$f = -\frac{1}{2}\,\widetilde{\mathrm{Re}}\left(\rho_1 U_1\,\frac{dU_1}{dp} + \bar{k}\right)p'. \tag{5}$$

In order to determine f according to (5), it is necessary to know the derivative dU_1/dp. For this, velocity U_1 is approximated as follows:

$$U_1 = U_{1k} + A\,(p - p_k) + B\,(p - p_k)^2.$$

Coefficient A is determined from the equality $A = (dU_1/dp)_k$. On the basis of (5),

$$\left(\frac{dU_1}{dp}\right)_k = -\bar{k}\,\frac{1 - U_{1k}^2}{p_k U_{1k}}\,,$$

since $p' \to -\infty$ as $x \to x_k$. Coefficient B is found by the method of least squares. The magnitude of τ can also be found by another and much more complicated method, namely by integrating the flow equation from the wall to some coordinate y. Calculations have shown that the results obtained in this way do not differ much from those obtained according to (3). Values of τ based on the approximation (3) are shown in Fig. 1 versus the y-coordinate. For illustration, the profiles of shearing stress τ have been plotted here at three sections near the pipe exit (in [1] the distance from the entrance to the exit was $x_k = 81.2$). From the diagram in Fig. 1 it is easy to determine the value of $\int_0^1 \tau dy$. Calculations show that this mean value of τ decreases while the frictional stress increases toward the exit. Furthermore the profiles of shearing stress, unlike in the case of an incompressible fluid, are appreciably distorted by negative pressure gradients. In order to determine the ε_T profile according to (2), now, it is necessary to approximate the velocity profile. Taking note of the test results in [1], one may, with sufficient confidence, assume the power law

$$u=V\left(\frac{y}{y_v}\right)^n \tag{6}$$

to be valid within a certain interval $\delta_b \leq y \leq y_V$, where exponent n is determined experimentally. Near the pipe axis, considering that $(\partial u/\partial y)_1 = 0$, we let

$$U=U_1+U_2r^2, \tag{7}$$

where U_1 and U_2 are determined from the smoothness conditions:

$$u\big|_{y=y_v}=U\big|_{y=y_v},\quad \frac{\partial u}{\partial y}\bigg|_{y=y_v}=\frac{\partial U}{\partial y}\bigg|_{y=y_v},\quad \frac{\partial^2 u}{\partial y^2}\bigg|_{y=y_v}=\frac{\partial^2 U}{\partial y^2}\bigg|_{y=y_v}.$$

These conditions yield expressions for U_1, U_2, and y_V in terms of a single parameter:

$$U_1=\frac{V}{2y_v},\quad U_2=\frac{n(1-n)}{2y_v^2}V,\quad y_v=\frac{1-n}{2-n}. \tag{8}$$

The profile according to (7) applies to $y_V \leq y \leq 1$.

Within the interval $0 \leq y \leq \delta_b$ at the wall, which will be called the boundary layer, the profile is approximated as follows:

$$u_n=\tau_0 y+ay^2+by^3. \tag{9}$$

Coefficient a is determined to (4) on the basis of the equality

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_0=\left(\frac{\partial \tau}{\partial y}\right)_0,$$

which follows from (2) on the premise that $(\partial\mu/dy)_0 = (\partial\varepsilon_T/dy)_0 = 0$. In order to determine coefficient b and the boundary-layer thickness δ_b, one uses the smoothness conditions at $y = \delta_b$:

$$u=u_n,\quad \frac{\partial u}{\partial y}=\frac{\partial u_n}{\partial y}.$$

From here we find without difficulty that

$$\left(\frac{1}{\delta_b}\right)^{1-n}=\frac{2\tau_0+a\delta_b}{3-n}\cdot\frac{y_v^n}{V}.$$

This equation is easily solved for δ_b by the method of successive approximations, assuming first $\delta_b = 0$ on the right-hand side. Having determined δ_b from (10), we then find coefficient b. Calculations have shown that δ_b is a rather small quantity (of the order of 10^{-3}) and decreases toward the pipe exit.

Thus, the proposed three-layer velocity profile involves two parameters: exponent n and velocity V. With the value of n based on the test data in [1], the value of V at any pipe section will be found from the condition of conserving the mass flow rate G and from the pressure p, both also known from tests:

$$\frac{G}{2p}=\int_0^{\delta_b}\frac{u_n}{1-u_n^2}(1-y)\,dy+\int_{\delta_b}^{y_v}\frac{u}{1-u^2}(1-y)\,dy+\int_{y_v}^{1}\frac{U}{1-U^2}(1-y)\,dy.$$

Knowing the τ and u profiles at any pipe section from (2), we obtain

$$\varepsilon_\tau=\frac{\tau}{\dfrac{\partial u}{\partial y}}-\mu.$$

The viscosity μ is assumed to vary with temperature according to the Sutherland equation. At Θ = const

$$T=1-v^2,$$

with $v = u_n$, u, and U within the respective zones along the transverse coordinates.

Considering that the stagnation temperature $\Theta \cong 288$°K, it is not difficult to derive the following relation between u and v:

$$\mu=\frac{(1-v^2)^{1.5}}{1-0.7024\,v^2}.$$

Calculated results for ε_T as a function of y are shown in Fig. 1b, c. The ε_T profiles farther away from the wall are shown in Fig. 1b. Evidently, the turbulent viscosity is qualitatively the same function of the transverse coordinate as in the case of an incompressible fluid [3, 4]. Unlike in the latter, however, in this case the maximum value of ε_T decreases as the magnitude of the pressure gradient increases nearer to the wall, while the frictional stress increases at the same time. Characteristically, the magnitude of the maximum ε_T remains lower than that for an incompressible fluid. It appears, furthermore, that an increasing pressure gradient reduces ε_T most significantly at a distance $y \approx 0.2$ from the wall. Nearer to the wall ($y \approx 0.05$) ε_T remains almost independent of the pressure gradient and at $y < 0.05$ ε_T appears as an inverse, though weak, function of p'. This may possibly be due to the inaccurate approximation of the velocity profile at the wall. In the boundary layer, to be sure, ε_T also increases with increasing $|-p'|$ (Fig. 1c). This diagram indicates also that in the boundary layer ε_T is much smaller than the dynamic viscosity μ even at high pressure gradients, the dynamic viscosity here being close to unity. Furthermore, the thickness of the viscous sublayer becomes apparently smaller ($\varepsilon_T \approx 0$) toward the pipe exit. This agrees with the results in [5].

It is well known that in a critical laminar gas flow the friction coefficient increases as the critical mode is approached [6]. This means that a negative pressure gradient, as has been found by the external-problem analysis, causes an increase in skin friction. In our case of turbulent flow the skin friction increases, although the turbulent viscosity decreases on the average. We thus have evidence that this effect is due primarily to the pressure gradient.

NOTATION

G	is the mass flow rate;
k	is the isentropic exponent;
M	is the Mach number;
p	is the pressure;
$Re = (\rho u_{lim}/\mu) \cdot (r_0/2)$;	
r_0	is the pipe radius;
u	is the longitudinal velocity;
x	is the longitudinal space coordinate;
y	is the transverse space coordinate;
$r = 1 - y$;	
δ_b	is the boundary-layer thickness;
ε_T	is the turbulent viscosity;
Θ	is the stagnation temperature;
μ	is the dynamic viscosity;
τ	is the shearing stress.

Subscripts

0	refers to pipe wall;
1	refers to pipe axis;
k	refers to pipe exit section;
lim	refers to limiting value (of velocity).

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